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Uni. Roll No.

Program: B.Tech. (Batch 2018 onward)

Semester: 4th

Name of Subject: Mathematics-111

Subject Code: BSCE- 101

Paper ID: 16180

Scientific calculator is Not Allowed

MORNING
19 SEP 2022

Time Allowed: 03 Hours

Max. Marks: 60

NOTE:

- 1) Parts A and B are compulsory
- 2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
- 3) Any missing data may be assumed appropriately

Part – A

[Marks: 02 each]

Q1.

- a) Write the negation of the disjunction:
Ram is in class XI or Arun is in class XII.
- b) State and prove linear property of Laplace Transform.
- c) Write the complex form Fourier integral .
- d) Define a group.
- e) State Modulation theorem of Fourier Transform.
- f) Define Lattice with one example.

Part – B

[Marks: 04 each]

Q2. Evaluate the following:

(i) $L^{-1}\left(\frac{1}{s^2(s^2-a^2)}\right)$ (ii) $L^{-1}\left(\log \frac{s+1}{s-1}\right)$

Q3. Evaluate the following:

(i) $L(\cos^3 2t)$ (ii) $L(t\sqrt{1+\sin t})$

Q4. Find Fourier sine integral of $f(x) = e^{-\beta x}$.

Q5. Construct a truth table for the compound proposition $\sim(p \vee q) \vee (\sim p \wedge \sim q)$.

- Q6. If $A = \{1, 2, 3, 4\}$ and consider the relation
 $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 2), (3, 3), (4, 2), (4, 3), (4, 4)\}$
 Is R a partial order relation? Justify your answer.

- Q7. Prove that the fourth roots of unity $1, -1, i, -i$ form an abelian multiplicative group.

Part – C

[Marks: 12 each]

- Q8. (a) Prove that the necessary and sufficient condition for a non-empty subset H of a Group $(G, *)$ to be a subgroup is $a \in H, b \in H \Rightarrow a * b^{-1} \in H$. (6)

- (b) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. (6)

OR

- (a) Prove that in a distributive Lattice, if an element has a complement then this complement is unique. (6)

- (b) Prove that the product of two Lattices is a Lattice. (6)

- Q9. Solve the following differential equation using Laplace transform:

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t, \quad x(0) = 0, \quad x'(0) = 1.$$

OR

Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x > 0, t > 0$, subject to the conditions $u_x(0, t) = 0$, $u(x, t)$ is bounded and $u(x, 0) = \begin{cases} x; & 0 \leq x \leq 1 \\ 0; & x > 1 \end{cases}$.
